# Quiz 12

March 3, 2017

Show all work and circle your final answer.

1. Evaluate the following series (if the sum does not exist, write DNE):

(a) $\sum_{n=2}^{\infty}(.7)^n = \frac{a}{1-r} = \boxed{\frac{.49}{1-.7}} = \boxed{\frac{49}{30}}$

$r = .7 < 1$, so converges

$a = (.7)^2 = .49$

(b) $\sum_{n=1}^{\infty}\left(\frac{3}{2}\right)^n$ $\boxed{\text{DNE}}$

$r = \frac{3}{2} > 1$, so diverges

(c) $\sum_{n=0}^{\infty}\left(\frac{3n+82}{41-n}\right)$ $\boxed{\text{DNE}}$

$\lim_{n\to\infty}\left(\frac{3n+82}{41-n}\right) = \frac{3}{-1} \neq 0$, so diverges by test for divergence

(d) $\sum_{n=0}^{\infty}\frac{(-2)^n}{x^n}$ for $|x| > 2$ $= \frac{a}{1-r} = \boxed{\frac{1}{1-\left(\frac{-2}{x}\right)}} = \boxed{\frac{x}{x+2}}$

$|r| = \left|\frac{-2}{x}\right| < 1$ if $|x| > 2$

$r = \frac{-2}{x}$

$a = 1$

2. For what values of $x$ does the series $\sum_{n=0}^{\infty} \left(\frac{x-1}{5}\right)^n$ converge?

$r = \frac{x-1}{5}$, so the series converges if

$-1 < \frac{x-1}{5} < 1$

$-5 < x-1 < 5$

$\boxed{-4 < x < 6}$

3. We use the substitution $x = \frac{2}{5}\sec\theta$ to rewrite $\int \frac{\sqrt{25x^2-4}}{x}\,dx$

as $\int \frac{2\tan\theta}{2/5\sec\theta}(2/5\sec\theta\tan\theta)\,d\theta$. Evaluate the second integral.
Leave your answer in terms of $\theta$.

$\int \frac{2\tan\theta}{\cancel{2/5}\,\cancel{\sec\theta}} \cdot \cancel{2/5}\,\cancel{\sec\theta}\tan\theta\,d\theta$

$= 2\int \tan^2\theta\,d\theta$

$= 2\int \sec^2\theta - 1\,d\theta$

$= \boxed{2[\tan\theta - \theta] + C}$